# Optimal solution method for Transportation problems of multiple variables

Dr. Brijendra Tiwari

(Head of Department, Mathematics, RKDF University, Bhopal)

Prof. Aarti Borasi,

Govt. JNS, P.G, College, Shujalpur M.P

***Abstract:*** *Transportation problem is considered a vitally important aspect that has been studied in a wide range of operation including research domain. As such it has been used in simulation of several real life problems. The transportation problem is a special class of model. It deals with the situation in which a commodity from several sources is shipped to different destinations with the main objective to minimize the total shipping cost .optimizing transportation problem of variable has remarkably been significant to various disciplines .It this paper , multiple variable will be optimized to reduce from-transportation cost using multiple method which will include.*

*Northwest corners method, least cost method, Vogel method and mode method. This will mainly aim at finding the best and cheapest route on how supply will be used to satisfy demand at specific points,*

**Key word;** *optimization techniques, transportation problem, northwest corner, least cost, Vogel, model.*

## I. INTRODUCTION

The transportation problem is one of the subclasses of L.P.P in which the objective is to transport various quantities of a single homogeneous commodity , that are initially stored at various sources to different destinations in such a way that the total transportation cost is a minimum. To achieve this objective we must know the mount and location of available supplies and the quantities demanded. It addition, we must know the costs that result from transporting one unit of commodity from various origins to various destinations

It this paper , transportations problem will be formulated as linear programming problems that will be solved using four method .The mode method is considered as being a standardized technique when it comes to obtaining optimal solution, on the other hand Vogel is believed to be an appropriate method . It usually tends to produce an optimal or near optimal initial solution. Several researches in this field determined that Vogel produce an optimum solution in almost 80% of this problem under test, Advantage of this method is that it accounts for its allocations.

Method of finding optimal solution to the transportation problem involve the following two step;

(1) Finding an initial basic feasible solution
(2) Perform an optimality test and iterating towards optimal solution is obtained.

In addition to the above two method the least cost method achieves its goals giving more allocations to the least cost cell. Rows and columns with complete allocations are ignored as the allocation process continues, this procedure is considered to be complete on condition that all requirements for all row and columns have satisfactorily been addressed. The northwest corner is the fourth method and it begins its allocation at the northwest corner of the matrix. It ensures that it assigns more units to each cell while putting into consideration to meet the requirement of not having more that m+n-1 filled cells. It this use

M= number of rows while

N= number of columns this procedure is iterated foe-the remaining rows until when the requirements for all row and columns will have met.

## II. TRANSPORTATION METHOD

When transportation method is employ in solving a transportation problem, Transportation model deal with problem concerning as to what happens to the effectiveness function when we associate each of a number of origins (sources) with each of a possibly different number of destinations (jobs), The total movement from each origin and the total movement to each destination is given and it is desired to find two the associations be made subject to the limitations on total. In such problem sources can be divided among the jobs or jobs may be done with a combination of sources, The distinct feature of transportation problems is that sources and jobs must be expressed in terms of only one kind of unit-The transportation, problem, very initial step that has to be under-taken is to obtain a feasible solution satisfying demand and supply require- meant several methods will be used in this paper to obtain this initial feasible problem. As mentioned earthier in this paper the method to be used will include.

## III. NORTHWEST CORNER METHOD

This is a method used to compute feasible solution of a transportation problem. In this method the basic variables are usually chosen from the left corner commonly referred following step are following steps are followed to obtain this feasible solution

(i) Start in the north-west ( upper left) corner of the requirements table I e the transportation martin framed in step 1 and compare the supply of plant 1 ( call it 5 ) with the requirement of distribution center 1 (call it D1)

(a) It D1<S1 I.e., it the amount required at d1 is less than the number of units available at s1 set x11 equal to d1 find the balance supply and demand and proceed to Cell (1, 2) (i.e., proceed horizontally)

Table 1

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| | 1 | 2(6) | 3 | 11 | 7 | 6/0 |
| Plants | 2 | 1 (1) | 0 (0) | 6 | 1 | 1/0 |
| | 3 | 5 | 8 (5) | 15 (3) | 9 (12) | 10/5/2/0 |
| Requirement | | 7/1/0 | 5/0 | 3/0 | 2/0 | |

(B) if D1=S1 , set x11 equal to D1 compute the balance supply and demand and proceed to cell (2,2) (I.e. proceed diagonally) . Also make a zero allocation to the least cost cell in S1/D11

(C) If D1>S1 set x11 equal to S1 , compute the balance supply and demand and proceed to cell ( 2,1) ( I.e. proceed vertically)

(ii) Continue in this manner step by step away from the north-west corner until finally a value is reached in the south-east corner

This in the present example (table-1) one proceeds as follows

(i) Set X11 equal to 6, namely the smaller of the amounts available at s1 (6) and that needed at d1 (7) and

(ii) Proceed to cell ( 2,1) (rule c) compare the number of units available at S2 (namely 1) with the amount required at d1 (i) and accordingly set X21=1 also sex x22=0 as per rule (b) above

(iii) Proceed to cell (3, 2) (rule b). now supply from plant S3 is 10 units while the demand for D2 is 5 unit accordingly set X32 equal to 5

(iv) Proceed to cell ( 3,3) (rule a) and allocate 3 there

(v) Proceed to cell ( 3,4 ) (rule a ) and allocate 2 there

It can be easily seen that the proposed solution is a feasible solution since all supply and demand constraints are fully satisfied .the following points may be noted in connection with this method.

(i) The quantities allocated are put in parenthesis and they represent the values of the corresponding decision variables. These cells are called basic or allocated or occupied or loaded cell, cells without allocations are called non-basic or vacant or empty or unoccupied or unloaded cell values of the corresponding variables are in the cell.

(ii) This method of allocation does not take into account the transportation cost and therefore may not yield a good (most economical) initial solution. The transportation cost associated with this solution is

Z=Rs. {2x6+1x1+8x5+15x3+9x2} x100
Rs. 11,600

## IV. ROW MINIMA METHOD

This method consists in allocating as much as possible in the lowest cost cell of the first row so that either the capacity of the first
Plant is exhausted or the require meant at distribution centre is satisfied are both In case of tie among the cost select arbitrarily .There cases arise

(i) If the capacity of the first plant is completely exhausted cross off the first row and proceed to the second row

(ii) If the requirement at distribution centre is satisfied cross off the column and reconsider the first row with the remaining capacity

(iii) It the capacity of the first plant as well as the requirement at distribution centre are completely make a zero allocation in the second lowest cost cell of the first row cross off the row as well as the column and move down to the second row

Continue the process for the resulting reduced transportation table all the rim conditions (supply) and requirement conditions) are satisfied

Table – (2)

Distribution centre

| plants | | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|---|
| | 1 | 2(6) | 3 | 11 | 7 | 6/0 |
| plants | 2 | 1 | 0 0 | 6 | 1 | 1/0 |
| | 3 | 5(1) | 8(4) | 15 (3) | 9 | 10/9/5/2/0 |
| | | | | | | |
| requirement | | 7/1/0 | 5/4/0 | 3/0 | 2/0 | |

The transportation cost associated with this solution is

Z=Rs {2x6+0x1+5x1+8x4+15x3+9x2}x100
=Rs 11,200

Which is less than the cost associated with solution obtained by N-W corner method

(3) **Column Minima Method**

This method consists in allocating as much as possible in the lowest cost cell of the first column so that either the demand of the first distribution centre is satisfied or the capacity or the plant is exhausted or both. In case of tie among the lowest cost cells in the column select arbitrarily three cases arise

(i) If the requirement of the first distribution centre is satisfied, cross off the first column and move right to the second column
(ii) If the capacity of plant is satisfied cross off row and reconsider the first column with the remaining requirement.
(iii) If the requirement of the first distribution centre as well as the capacity of the plant are completely satisfied made a zero allocation in the second lowest cost cell of the first column.
Cross off the column as well as the row and move right to the second column.

Continue the process for the resulting reduced transportation table until all rim conditions are satisfied.

Table -3

Distribution centers

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| | 1 | 2(6) | 3 | 11 | 7 | 6/0 |
| Plants | 2 | 1(1) | 0 | 6 | 1 | 1/0 |
| | 3 | 5(0) | 5(5) | 15(3) | 9(2) | 10/9/5/2/0 |
| Requirement | | 7/6/0 | 5/4/0 | 3/0 | 2/0 | |

The transportation cost associated with this solution is

Z=Rs {2x6+1x1+5x0+8x5+15x3+9x2} x100 =Rs 11,600

Which is same as the cost associated with solution obtained by n w corner method

**(4) Least – Cost method Core Matrix minima method or lowest cost entry method)**

This method consists in allocating as much as possible in the lowest cost cell I cells and them further allocation is done in the cell/cells with second lowest cost and so on in case of tie among the cost select the cell where allocation of more number of units can be made. Consider the matrix for the problem under study

TABLE -4

Distribution Centers

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| Plants | 1 | 2(6) | 3 | 11 | 7 | 6/0 |
| | 2 | 1 | 0(1) | 6 | 1 | 1/0 |
| | 3 | 5(1) | 8(4) | 15(3) | 9(2) | 10/9/5/3/0 |
| Requirement | | 7/1/0 | 5/4/0 | 3/0 | 2/0 | |

Here the lowest cost-cell is (2, 2) and maximum possible allocation (mating supply and requirement positions) is made here. Evidently .maximum feasible allocation in cell (2, 2) is (1). This meets the supply position of plant 2 .Therefore row 2 is crossed out indicating that no allocations are to be made in cell ( 2,1) (2,3) and (2,4).

The next lowest cost cell (excluding the cells in row 2) is (1, 1) maximum possible allocation of (6) is made her and row 1 is crossed out. Next lowest cost cell in row 3 in (3, 1) and allocation of (1) is done here. Likewise allocation of (4), (2) and (3) are done in cell (3, 2) (3, 4) and (3, 3) respectively. The transportation cost associated with this solution is

Z=Rs (2x6+0x1+5x1+8x4+15x3+9x2) x100 =Rs 11,200

Which is than the cost associated with the solution obtained by n-w corner method

**(5) Vogel's approximation method (VAM) or penalty method or regret method.**

This method tends to repeat procedure to compute a basic feasible solution of the transportation problem .However some step have to be followed while carrying out such computations .This include

(i) Two cell have to be identified offer cell identification the difference between these two values has to be determined and written against the corresponding row on the side of the table .The procedure has to be repeated for column whereby the cell with the minimum transportations cost as well as the cell with the next to minimum transportations cost have to be identified in each column and their difference determined so that it can be written against the corresponding column.

(ii) After this next step is to determine the minimum difference so that in case it is located on the side of the table , maximum allotment is granted to the cell with minimum cost of trans- potation in that particular column. In case the corresponding difference for two or more rows / columns are termed as being equal the row on the very upper part of the table and the far left column should be selected. This can be illustrated best as shown below in table 5

Table - 5

Distributioncenters

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| Plants | 1 | 2(1) | 3(5) | 11 | 7 | 6/1/0{1}{1}< |
| | 2 | 1 | 0 | 6 | 1(1) | 1/0{1} |
| | 3 | 5(6) | 8 | 15(3) | 9(1) | 10/4/3/0{3}{3}{4} |
| Requirement | | 7/6/0 | 5/0 | 3/0 | 2/1/0 | |
| | | {1} {3} | {3} {5} | {5} {4} | {6} {2} | |
| | | {3} | | {4} | {2} | |

The cost of transportation associated cost- the about solution is

Z=Rs(2x1+3x5+1x1+5x6+15x3+9x1) x100

=Rs (2+15+1+30+45+9)x100 =Rs 10,200

Which is evidently the least of all the values of transportations cost found by different methods. Since Vogel's Approximation method results in the most economical initial feasible solution we shall use this initialfeasible solution we shall use this method for finding such a solution for all transportation problem hence forth.

Just like with the previous method Vogel method also managed to satisfy supply and demand requirements after obtaining initial solution using their first there method it has to pass through an optimal lestins process

By definition an optimal solution refers to a solution in which no more t+

Transportationroutes can manage to reduce the total Transportation cost.

This implies that the process of evaluating cells that are not occupied in a transportation table has to be initiated. This process is usually carried outin terms of opportunities aiming at reducing the overall transportations cost.

The new set of transportation routes must them include a cell having the largest negative opportunity cost but remain unoccupied .In such cases the sixth method mentioned is considered to be the most efficient this method is the MODE method .

## V. MODI METHOD

THE MODI (modified distribution) method allows us to compute improvement quickly for each unused squarewithout drawing all of the closed paths because of this it can often provide considerable time provide considerable time savings over other method for solving transportation problem

MODE provides a new means of finding the unused route with the target negative improvement index once the largest index is trace only one closed path .This path helps determine the maximum number of units that can be shipped via the best unused route.

How to use the Modi method

In applying them MODI method we begin with an initial solution obtained by using the north west corner rule or any other rule but now we must compute a value for each row (call the value R1 R2 R3 in there are three rows and for each column ( K1 K2 K3) in the transportation table ,In general WE let

Ri= value assigned to row i

Kj= value assigned to column j

Cij=cost in square it (cost of shipping from source i to destination j)

The Mode method then requires live step;

(1) To compute the values for each row and column set

Ri+Kj=Cij

But only for those squares that are currently used occupied for example .If the square at the intersection of row 2 and column is occupied we set

R2+K1=C21

(2) After all equation have been co-written set r1=0
(3) Solve the system of equation for all R and K values

(4) Compute the improvement index for each unused square by the formula improvement index

(Iij)= (ij -Rj-k

Select the target negative index and proceed to solve the problem as you did using the stepping - stone method

Table -6

Distribution centers

| | | | | | | Supply |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3(5) | 11(1) | 7 | 6 |
| Plants | 2 | 1 | 0 | 6(1) | 1 | 1 |
| | 3 | 5(7) | 8 | 15(1) | 9(2) | 10 |
| Requirement | | 7 | 5 | 3 | 2 | |

For this allocation matrix the transportation cost is

Z= Rs{5x7+3x5+11x1+6x1+15x1+9x2}x100 Z= Rs 10,000

## VI. CONCLUSION:

This paper considered optimization techniques of transportation problem for four variables using four method optimization of problem is the same as choosing optimal solution from the available alternativeasevidenced from previous researches this technique is applicable in awide range of fields. However MODI method employed in this paper can be used with good deal of success in solving such problems .This is because while working hand in hand with the remaining live method, MODI method computed the optimal solution within a shorter period of time. Beside it reduced complex ties v.a producing a simple and clear solution that could be easily used in other areas for optimizing other problems

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