ANALYSIS OF MIMO SYSTEM UNDER DIFFERENT SIGNALING CONDITION FOR WIRELESS COMMUNICATION SYSTEM

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Abstract

The wireless communication depends on the quality of service and high-speed data rate. The MIMO (multiple input multiple output) creates the diverse space of the wireless communication system. The MIMO system used different types of signalling methods for modulation and sampling for transmission. The performance of the MIMO system depends on various parameters such as channel capacity, fading of signal, sampling of frequency, beamforming, and reuse of channel. The massive MIMO system is increasing rate of the base station and a number of array of antennas. The balanced and unbalanced MIMO condition impact on the performance of the MIMO system. The major issue and challenge are minimization of noise and channel error for the on-air transmission medium. The process of fading also decreases the performance of MIMO System. This dissertation analysed the various condition of noise and error estimation technique such as ZF (zero forcing) is an ideal condition for transmission of signals. The minimization of noise and signals error increase the gain value of signals. For the validation of signalling process used MATLAB software and evaluated standard parameters.

Keywords: - Wireless Communication, Massive MIMO, MIMO, ZF, MSME, BER, Interference, Throughput

Introduction

The Upcoming Generation of Wireless communication design and dedicated to increasing capacity of the network in the process of enhanced users' behaviors and applicability like real-time data streaming. There is various approach to improve network capacity, such as multiple input multiple output, frequency reuse, massive MIMO system and reduction of interference in local communication models[4, 5]. MIMO limit results are known for some cases, where the comparing multiuser issues stay unsolved. Specifically, next to no is thought about multiuser limit without the supposition of amazing channel state information at the transmitter (CSIT) and at the recipient (CSIR). While there stay many open issues in getting the single-client limit under broad presumptions of CSI and CDI, for a few intriguing cases the arrangement is known. This segment will give an outline of known outcomes for single-client MIMO channels with specific spotlight on extraordinary instances of CDI at the transmitter, just as the beneficiary[26].

Precoding is a potential technique to overcome the detection complexity issue at the downlink receivers. Based on the CSI at the transmitter, precoding provides high capacity and full transmit diversity. Meanwhile, the co-channel interference (CCI) among antennas/users can also be completely eliminated, the signal flows at different antennas/users therefore can be orthogonal, which enables low-complexity detection at the receiver and also empowers the application of space division multiple access (SDMA) in multi-user MIMO (MU-MIMO) systems[28]. Thus, precoding has been considered as one of the core technologies for the downlink massive MIMO systems. Previous efforts to combine precoding with SM in single carrier MIMO systems have been well discussed in point-to-point and broadcast channels, which select the receiver-side active antennas to carry information and are reported to have improved performance relative to conventional pre-coded single carrier MIMO schemes. The above-mentioned works exploit the space domain degree of freedom to perform index modulation and indicate a promising future of the PIM schemes.

Literature Survey

Miranda, Ricardo Kehrle Et al. [4] In request to completely abuse the scant range, radio wire exhibits are consolidated into remote specialized gadgets in 4G and 5G correspondence organizations to send MIMO-OFDM frameworks. As of late, the least squares Khatri–Rao factorization has been applied to MIMO-OFDM frameworks for semi-daze joint channel and image assessment. Its cubic computational intricacy is restrictive when the quantity of communicate and get radio wires is exceptionally enormous. Hence, the normal vector and Hadamard proportion rank one estimation has been talked about for MIMO-OFDM frameworks, demonstrating a direct unpredictability, however being restricted to channels and communicated images with balances. creators present four novel MIMO-OFDM calculations for monstrous reception apparatus exhibit frameworks that outflank the cutting-edge approaches regarding unpredictability as well as exactness. The four talked about plans are the rotating least squares with vector choice introduction, the vector projection rank one estimate including vector choice position one instatement, the factorization dependent on eigenvalue disintegration with eigenvector projection and the factorization dependent on sectional shortened particular worth decay and vector projection. their scientific multifaceted nature investigation and mathematical outcomes prove the compromises offered by the diverse recipient calculations as far as intricacy, parallelism and execution.

Al-Askery, Ali J. Et al. [6] creators determine the digit mistake rate and pairwise blunder likelihood (PEP) for MIMO-OFDM frameworks for various Mary regulations dependent on the rough commotion appropriation after channel balance. The PEP is utilized to acquire the upper-limits for convolutionally coded and turbo coded huge MIMO-OFDM frameworks for various code generators and get receiving wires. Moreover, intricacy examination of the LLR values is performed utilizing the estimated commotion likelihood thickness work. The inferred LLR calculations can be tedious when the quantity of get radio wires is enormous in monstrous MIMO-OFDM frameworks. Subsequently, a decreased intricacy estimate is presented utilizing Newton's addition with various polynomial requests and the outcomes are contrasted and the specific recreations. The Neumann enormous network guess is utilized to plan the beneficiary for a zero-compelling equalizer by lessening the quantity of tasks needed in figuring the channel framework backwards. Reproductions are utilized to exhibit that the outcomes acquired utilizing the determined conditions coordinate intently the Monte Carlo re-enactments.

Araújo, Daniel Costa Et al. [7] Creators have talked about new calculations to appraise the CSI in monstrous MIMO-OFDM frameworks that mutually abuse the sparsity and multidimensional nature of the channel. The talked about tensor-based assessors empower the UEs to gauge the MIMO channel by accepting that the BS and UE have a set number of RF chains, the MIMO channel is recurrence specific, and the framework has restricted pilot assets for channel securing, permitting to lessen the preparation overhead. The presentation of the talked about T-OMP-JS and T-OMP-SS can be additionally improved to manage off-framework issues, where the channel ways don't match to the premise set (AF, AT, AR). This circumstance occurs, for example, when there is vulnerability in the structure of the exhibit reaction vectors at the transmitter or potentially beneficiary because of alignment mistakes. For this situation, tensor-based word reference learning calculations could be utilized.

Elwekeil, Mohamed Et al. [18] A profound learning approach for versatile regulation and coding in down to earth MIMO-OFDM frameworks is introduced. The talked about methodology utilizes a profound convolutional neural organization that is prepared utilizing the assessed channel coefficients and standard deviation as the information highlights. The examined approach dependably predicts the proper adjustment and coding plan even within the sight of reasonable weaknesses, for example, flawed planning synchronization, transporter recurrence counterbalance and channel assessment at the MIMO-OFDM beneficiaries. Recreation results uncover that the examined approach beats both k-NN, SVM and DNN approaches regarding the bundle blunder rate and the framework throughput.

. Fundamental of MIMO System

The multiple signal processing techniques applied on MIMO system. Signal processing methods may be divided into linear and non-linear methods. In linear detection methods the channel output is mapped into its output using a linear transform[11, 13]. This section is concerned with the effectiveness of linear receive combining methods in MIMO implementations. This section looks at four linear receive combining methods; after a briefly description of the signal model used, we will explain these combining methods in more detail[15].

The signal model describes as

$$y = Hx + n \qquad (1)$$

Where, H is the channel matrix, x is the transmitted signal vector, n is noise vector, and y is the received signal vector.

ZERO FORCING (ZF) EQUALIZATION

ZF is a least squares approximation method which is use to approximate the solution for an overdetermined linear system where there are more equations than there are variables. For massive MIMO, there are more antennas at the transmitter than there are at the receiver. The linear system that relates the received signals y, the transmitted signals x, and the channel matrix H is an overdetermined linear system. ZF seeks to approximate the transmitted signals from the received noisy signals. The method seeks to minimize the square of the error between the received and the transmitted signals. It is thus called a least squares method[17, 19]. From the signal model earlier, this means minimizing the norm

$$\|y - Hx\|^2$$

Finding the minimum means choosing x such that the differential of the norm with respect to x is zero.

$$\frac{d}{dx}\|y - Hx\|^2 = 0 \quad (2)$$

$$x^{-} = (H^T H)^{-T} H^T y \quad (3)$$

x^{-} is used here to show that it is an approximation of x. The quantity $(H^T H)^{-T} H^T$ is usually denoted H^{+} and is called the pseudo-inverse. The channel matrix is generally complex and is thus its pseudo-inverse is given by:

$$H^{+} = (H^H H)^{-1} H^H \quad (4)$$

where $(.)^H$ denotes the Hermitian transpose. Zero Forcing is simple to implement but it however has a problem of amplifying the noise. ZF needs high SNR values for it to perform well.

REGULARIZED ZERO FORCING (RZF)

Direct inversion of the channel matrix, as done in the ZF equalization method, may lead to poor results when the channel matrix rank is low. This is when the channel matrix becomes close to being singular and matrix inversion becomes impossible. The receiver will therefore not be able to distinguish between different MIMO paths. In order to mitigate this, the RZF method was designed. For RZF the inversion of the channel matrix is regularized by adding a scaled identity matrix to HH^H before matrix inversion[20, 22]. The receive combining vector as given is:

$$T_{RZF} = H^H(\beta I + HH^H)^{-1} \quad (5)$$

where β is the regularization factor?

MINIMUM MEAN SQUARE ERROR (MMSE)

The linear MMSE equalizer utilizes the Bayesian approach to estimating the transmitted signal vector x^{-} from the observed signal vector y at the receiver[23, 25].

The MMSE estimator is the matrix C in Eq. below that minimizes the mean of the square of the error vector between the estimated signal x^{-} and the transmitted signal x:

$$x^{-} = C^H y \quad (6)$$

The MMSE estimator seeks to minimize the objective function below:

$$x^{-} = argmin_c E\{\|x^{-} - x\|^2\} \quad (7)$$

$$= E\{\|C^H y - x\|^2\} \quad (8)$$

where E is statistical the expectation. The cross covariance and covariance matrices for x and y are respectively as:

$$R_{xy} = E(xy^H) \quad (9)$$

$$R_{yy} = E(yy^H) \quad (10)$$

Differentiating Eq. 9 with respect to and equating to C zero gives us the MMSE estimator as

$$C = R_{yy}^{-1} R_{yx} \quad (11)$$

Substituting $Hx + n$ for y in Eq.11 leads to:

$$R_{yy} = E((Hx + n)^H (Hx + n)) \quad (12)$$

Noting that y and the noise n are uncorrelated, and the expectation $E(nn^H)$ is the noise variance σ_n^2, the correlation R_{yy} in Eq.13 is given by:

$$R_{yy} = HR_{xx}H^H + \sigma_n^2 I \quad (13)$$

Where, I is an identity matrix, similarly:

$$R_{yx} = E(yx^H) \quad (14)$$

$$= HE(xx^H) \quad (15)$$

Taking consideration of the fact that noise is uncorrelated with the signal and assuming the unity for transmit power, and substituting R_{yy} and R_{yx} for expressions in Eq.14 and Eq.16 respectively into Eq.12 leads us to the final MMSE estimator C as given by as :

$$C = H^H(HH^H + \sigma^2 I)^{-1} \quad (16)$$

where σ^2 is the noise variance? At high SNR values, MMSE converges to ZF equalizer while at low SNR it converges to the matched filter equalizer.

Basic of Adaptive Model

Adaptive model is basically concept of frequency reuse concept in MIMO system. The concept of frequency reuse increases the channel capacity and throughput of communication channel. The description of adaptive model is described here.

1. Let $h_{m,n}$ be a complex number corresponding to the channel gain between transmit antenna n and the receive antenna m. If at a certain time instant adaptive signals $\{x_1, x_2, \ldots, x_{nt}\}$ are transmitted via the n_t antennas, the received signals at antenna m can be expressed as

$$y_m = \sum_{n=1}^{n_t} h_{m,n} x_n + e_m \quad .....(1)$$

where e_m is a noise term. The relation is easily expressed in a matrix framework. Let *x* and *y* be n_t and n_r vectors containing the transmitter and receiver data, respectively. Define the following n_r xn_t adaptive channel gain matrix:

$$H = \begin{bmatrix} h_{1,1} \ldots\ldots\ldots h_{1,nt} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ . \\ h_{nr,1} \ldots\ldots. h_{nr,nt} \end{bmatrix}$$

Then we have

y = Hx + e ....(2)

where e = $[e_1 \ldots\ldots. e_{nr}]^T$ is a vector of noise samples. If several consecutive vectors {x_1,..,x_N} are transmitted, the corresponding received data can be arranged in a matrix

2. We can express as adaptive signal as Y = [y_1 ...... y_N] ....(3)
3. We can write as

Y = HX + E...............................(4)

4. We can characterize channel selective as adaptive channel as follows:

$$H(z^{-1}) = \sum_{l=0}^{L} H_l z^{-l} \quad ....(5)$$

where H_l are the n_r xn_t adaptive channel matrices corresponding to the time delays *l* = 0, ..,L. Here the channel is assumed to have (L+1) number of taps. Also, L = 0 corresponds to channel fading model.

5. Now we can define selective of adaptive channel are

y = GX + e ....(6)

where x = $[x^T(-L) \; . \; . \; . \; x^T(N_0 + L - 1)]^T$

y = $[y^T(0) \; . \; . \; . \; y^T(N_0 + L - 1)]^T$

e = $[e^T(0) \; . \; . \; . \; e^T(N_0 + L - 1)]^T$

and

$$G = \begin{bmatrix} H_L H_{L-1} \ldots\ldots H_1 H_0 \, 0 \ldots\ldots\ldots\ldots\ldots\ldots 0 \\ 0 H_L H_{L-1} \ldots\ldots\ldots H_1 H_0 \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots. \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 0 \\ 0 \ldots\ldots\ldots\ldots H_L H_{L-1} \ldots\ldots\ldots\ldots H_1 H_0 \end{bmatrix}$$

6. Finally select the transmitted signal

Find the value of outage probability and BER in consideration of SNR value.

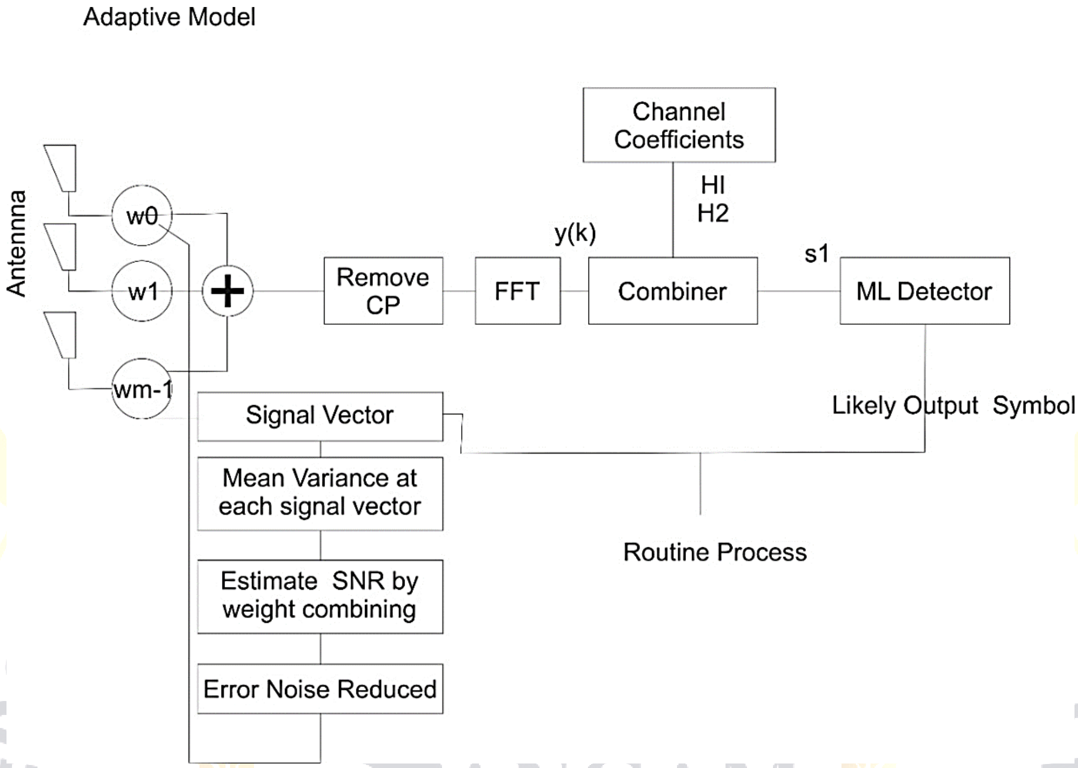

Figure 1: Process block diagram of adaptive model for the selection of frequency on the basic of adaptive model.

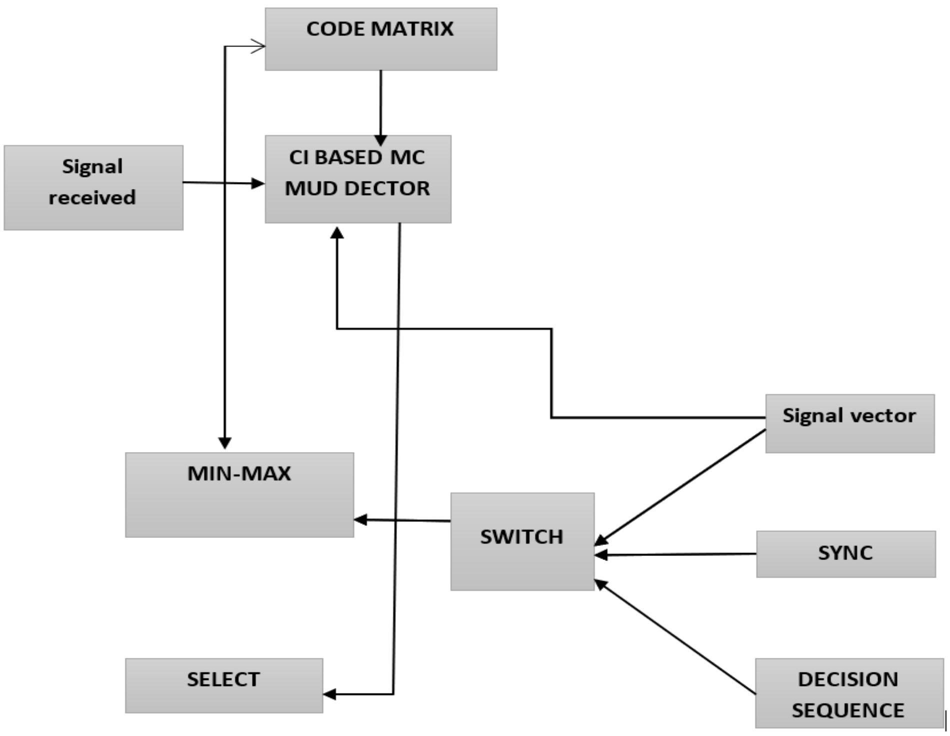

Figure 2: Process block diagram of adaptive model for the selection of weight value for sequence of signal selection.

Analysis

Parameter	Value

Number of BS or Cells	1 *and* 16
Pilot Reuse Factors	1,2,4
No.of UEs,K	20
No.of Antennas,M	20: 200
No.of Simulations runs	100
Samples per coherence block	200
UL transmit Power	20 *dBm*
Pathloss Exponent	3.76
Channel gain at 1 *Km*	−148.1*dB*
Shadow fading Standard Dev	10
Cell Area	1 *Km x* 1 *Km*

Table 1: Simulation parameters.

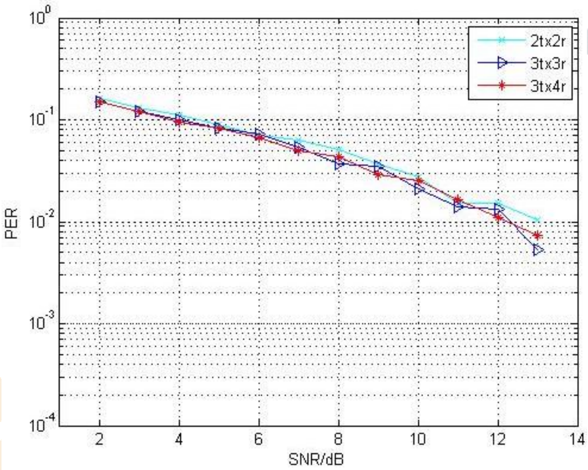

Figure 3: Analysis between PER (packet error rate) and SNR (Signal to noise ratio). Here performance consider on the basis of 2tx2r, 3tx3r, 3tx4r.

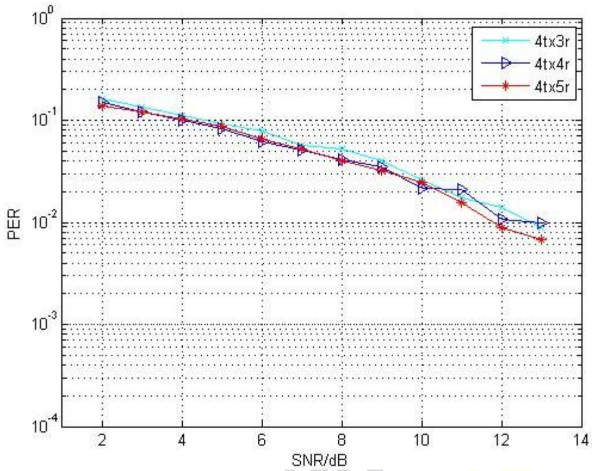

Figure4: Analysis between PER (Packet Error Rate) and SNR (Signal to Noise Ratio). Here performance consider on the basis of 4tx3r, 4tx4r, 4tx5r.

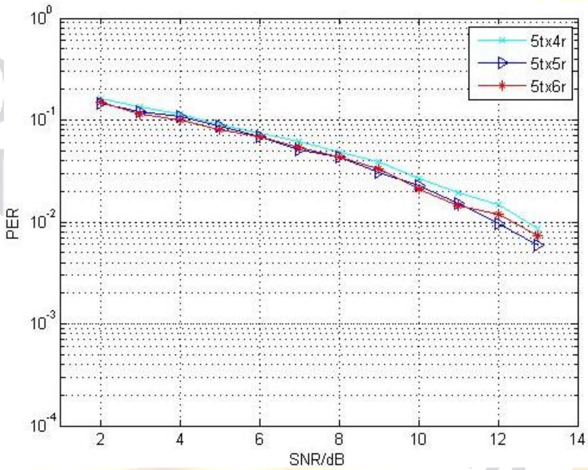

Figure 5: Comparative performance analysis between PER (Packet Error Rate) and SNR (Signal to Noise Ratio). Here performance consider on the basis of 5tx4r, 5tx5r, 5tx6r.

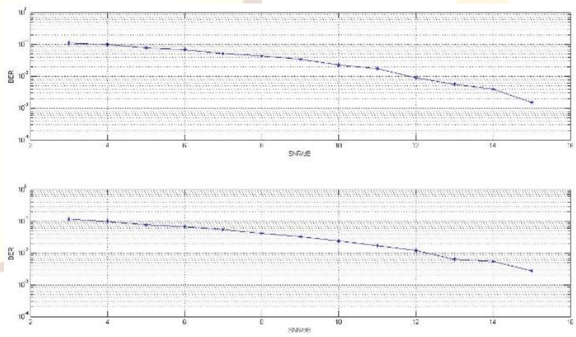

Figure 6: Analysis between BER (Bit Error Rate) and SNR (Signal to Noise Ratio). Here performance consider on the basis of 4, 6, 8, 10, 12, 14, 16.

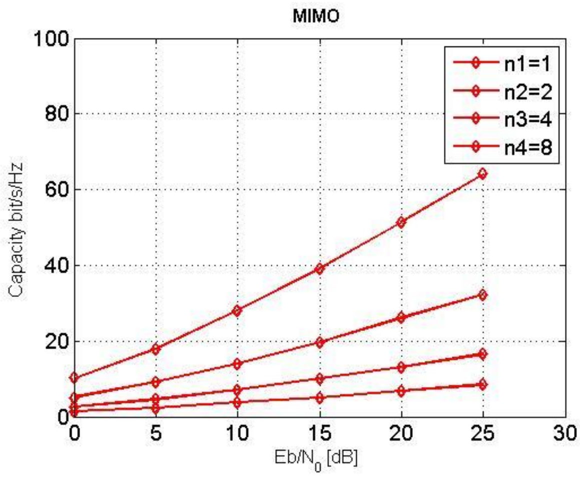

Figure 7: Analysis between capacity ($bits/Hz$) and SNR (Signal to Noise Ratio). Here performance consider on the basis of n1=1, n2=2, n3=4, n4=8.

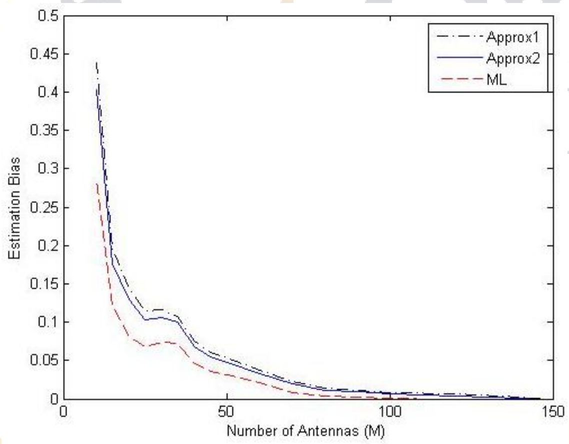

Figure 8: Analysis between estimation bias and number of antennas. Here number of antennas 10 to 150 and estimation bias 0.44 to 0.

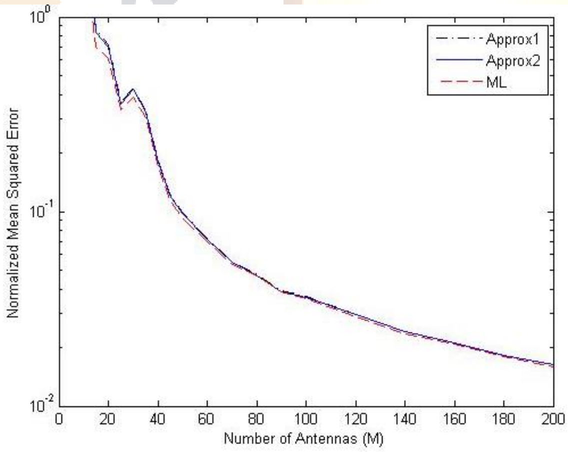

Figure 09: Analysis between NMSE (Normalized Mean Squared Error) and number of antennas. Here number of antennas 15 to 200 and NMSE 0 to 0.08.

Conclusion

The performance of MIMO system. The major analysis based on different combined methods such as MMSE, ZF, RZF and MR. the simulation results of correlated channel and uncorrelated channel indicates the situation of antenna installation on base station. In an offer to show a more practical proliferation channel, it was in this way esteemed important to do comparative reproductions for a Rayleigh associated fading channel. As was normal, the phantom proficiency execution for all sign preparing techniques was lower for the associated channel contrasted with the previous examined uncorrelated channel case. RZF and MMSE indicated similar patterns as were watched for the uncorrelated channel case. Up until the quantity of radio wire components was double the quantity of users, the unearthly effectiveness for MR was higher than that for ZF. After that specific point, ZF execution improved and moved toward both RZF and MMSE.

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